Week 8 Practical Session

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Structural equation models

There are two commonly used packages for doing structural equation modelling in R: `sem` and `lavaan`. I use the latter because I find the models easier to specify. It also includes some utilities to make creating covariance matrices a bit easier. First, enter it as a single string.

```
# input the correlations in lower diagonal form
houghtonLower.cor <- '
1.000
.668 1.000
.635  .599 1.000
.263  .261   .164 1.000
.290  .315   .247  .486 1.000
.207  .245   .231  .251  .449 1.000
-.206 -.182  -.195 -.309 -.266 -.142 1.000
-.280 -.241  -.238 -.344 -.305 -.230  .753 1.000
-.258 -.244 -.185  -.255 -.255 -.215  .554  .587 1.000
.080  .096  .094  -.017  .151  .141 -.074 -.111  .016 1.000
.061  .028 -.035  -.058 -.051 -.003 -.040 -.040 -.018 .284 1.000
.113  .174  .059   .063  .138  .044 -.119 -.073 -.084 .563  .379 1.000 '
```

Then use the function `getCov` to create a correlation matrix, and the function `cor2cov` to convert to a covariance matrix:

```
houghtonFull.cor <-
 getCov(houghtonLower.cor, names = c("wk1","wk2","wk3","hap","md1","md2","pr1","pr2",
                                     "app","bel","st","ima"))

houghtonFull.cov <-
 lavaan::cor2cov(houghtonFull.cor, sds = c(0.939, 1.017, 0.937, 0.562, 0.760, 0.524, 0.585, 0.609,
                                   0.731, 0.711, 1.124, 1.001))
print(houghtonFull.cov, digits = 1)
```

```
      wk1   wk2   wk3    hap   md1    md2   pr1   pr2    app    bel     st
wk1  0.88  0.64  0.56  0.139  0.21  0.102 -0.11 -0.16 -0.177  0.053  0.064
wk2  0.64  1.03  0.57  0.149  0.24  0.131 -0.11 -0.15 -0.181  0.069  0.032
wk3  0.56  0.57  0.88  0.086  0.18  0.113 -0.11 -0.14 -0.127  0.063 -0.037
hap  0.14  0.15  0.09  0.316  0.21  0.074 -0.10 -0.12 -0.105 -0.007 -0.037
md1  0.21  0.24  0.18  0.208  0.58  0.179 -0.12 -0.14 -0.142  0.082 -0.044
md2  0.10  0.13  0.11  0.074  0.18  0.275 -0.04 -0.07 -0.082  0.053 -0.002
pr1 -0.11 -0.11 -0.11 -0.102 -0.12 -0.044  0.34  0.27  0.237 -0.031 -0.026
pr2 -0.16 -0.15 -0.14 -0.118 -0.14 -0.073  0.27  0.37  0.261 -0.048 -0.027
app -0.18 -0.18 -0.13 -0.105 -0.14 -0.082  0.24  0.26  0.534  0.008 -0.015
bel  0.05  0.07  0.06 -0.007  0.08  0.053 -0.03 -0.05  0.008  0.506  0.227
st   0.06  0.03 -0.04 -0.037 -0.04 -0.002 -0.03 -0.03 -0.015  0.227  1.263
ima  0.11  0.18  0.06  0.035  0.10  0.023 -0.07 -0.04 -0.061  0.401  0.426
      ima
wk1  0.11
```

```
wk2  0.18
wk3  0.06
hap  0.04
md1  0.10
md2  0.02
pr1 -0.07
pr2 -0.04
app -0.06
bel  0.40
st   0.43
ima  1.00
```

Specifying models in `lavaan`

Models are specified in a single string, with each equation being specified on a new line within the string. Generally, we use the usual R formula syntax, but there are three common types of equation:

- **Measurement model** Equations specified using `=~` to separate lhs and rhs of equation. Lhs is the unobserved construct, rhs are the measured variables that act as its indicators.
- **Structural model** Equations specified using `~`. Variables on both sides can be measured (as in the path analysis example) or unobserved.
- **Variances and covariances** Equations specified using `~~`. This is usually used as a way of telling `lavaan` which of these are to be considered free parameters to be estimated.

You can constrain a parameter that would be free by default or free a parameter that would be constrained by default using *premultiplication*. To free a normally constrained parameter, premultiply by `NA`, eg,

```
Y =~ NA * X1 + X2
```

To constrain a parameter that would otherwise be free, premultiply by the value to which it is to be constrained, eg,

```
Y ~~ 1 * Y
```

To constrain two or more parameters to be equal, premultiply by the same label:

```
Y1 =~ a * X1 + X2
Y2 =~ a * X1 + X3
```

You can see the defaults in the help pages for the functions `cfa` and `sem`. For example,

"The `sem` function is a wrapper for the more general `lavaan function`, but setting the following default options: `int.ov.free = TRUE`, `int.lv.free = FALSE`, `auto.fix.first = TRUE` (unless `std.lv = TRUE`), `auto.fix.single = TRUE`, `auto.var = TRUE`, `auto.cov.lv.x = TRUE`, `auto.th = TRUE`, `auto.delta = TRUE`, and `auto.cov.y` = TRUE."

Example

This example comes from a study of job satisfaction among 263 university employees. The hypothesis is that constructive thinking reduces dysfunctional thinking, which leads to an enhanced sense of well-being, which in turn results in greater job satisfaction. The four theoretical constructs are:

1. Constructive (opportunity oriented) thinking
2. Dysfunctional (obstacle oriented) thinking
3. Subjective well-being
4. Job satisfaction

The structural part of the model reprsents the hypotheses that:

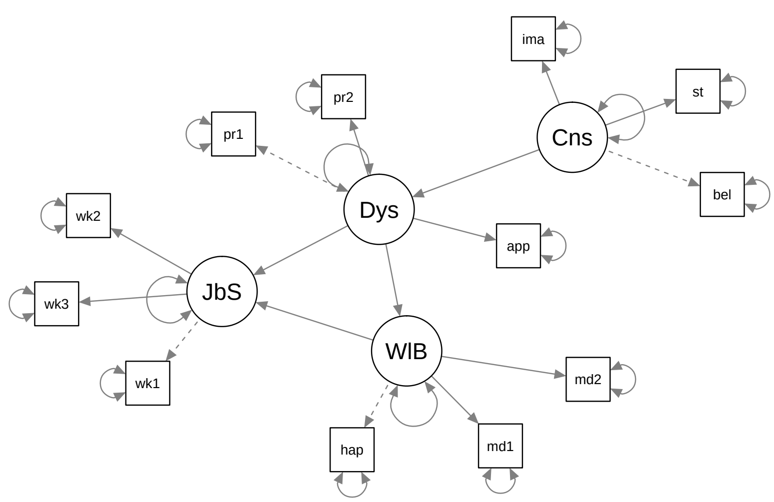
1. Both dysfunctional thinking and subjective well-being have direct effects on job satisfaction
2. Dysfunctional thinking has a direct effect on subjective well-being
3. Constructive thinking has a direct effect on dysfunctional thinking

In the measurement part of the model, each construct has three indicators (see p. 221 of Kline).

```
# specify sr model
houghtonSR.model0 <- '
# measurement part

Construc =~ bel + st + ima
Dysfunc =~ pr1 + pr2 + app
WellBe =~ hap + md1 + md2
JobSat =~ wk1 + wk2 + wk3

# structural part

Dysfunc ~ Construc
WellBe ~  Dysfunc
JobSat ~ Dysfunc + WellBe '

# fit sr model to data
srmodel0 <- sem(houghtonSR.model0,
         sample.cov = houghtonFull.cov,
         sample.nobs = 263)
```

This is what the model looks like as a diagramme, followed by the results. The function for producing these diagrams is `semPaths`, which is in the `semPlots` package.

```
semPaths(srmodel0, layout = 'spring')
```




```
summary(srmodel0, fit.measures = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  43 iterations

  Number of observations                           263
```

```
  Estimator                                         ML
  Minimum Function Test Statistic               66.313
  Degrees of freedom                                50
  P-value (Chi-square)                           0.061

Model test baseline model:

  Minimum Function Test Statistic             1087.490
  Degrees of freedom                                66
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.984
  Tucker-Lewis Index (TLI)                       0.979

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3126.092
  Loglikelihood unrestricted model (H1)      -3092.936

  Number of free parameters                         28
  Akaike (AIC)                                6308.185
  Bayesian (BIC)                              6408.205
  Sample-size adjusted Bayesian (BIC)         6319.432

Root Mean Square Error of Approximation:

  RMSEA                                          0.035
  90 Percent Confidence Interval          0.000  0.056
  P-value RMSEA <= 0.05                          0.866

Standardized Root Mean Square Residual:

  SRMR                                           0.045

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Construc =~
    bel               1.000
    st                1.060    0.178    5.941    0.000
    ima               1.861    0.331    5.627    0.000
  Dysfunc =~
    pr1               1.000
    pr2               1.126    0.080   14.108    0.000
    app               0.991    0.089   11.184    0.000
  WellBe =~
    hap               1.000
```

```
    md1                 1.768    0.242    7.305    0.000
    md2                 0.812    0.125    6.484    0.000
  JobSat =~
    wk1                 1.000
    wk2                 1.031    0.081   12.729    0.000
    wk3                 0.892    0.073   12.160    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Dysfunc ~
    Construc           -0.140    0.078   -1.804    0.071
  WellBe ~
    Dysfunc            -0.332    0.062   -5.382    0.000
  JobSat ~
    Dysfunc            -0.259    0.131   -1.983    0.047
    WellBe              0.907    0.220    4.124    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .bel                 0.289    0.043    6.669    0.000
   .st                  1.017    0.097   10.441    0.000
   .ima                 0.255    0.124    2.061    0.039
   .pr1                 0.105    0.016    6.702    0.000
   .pr2                 0.070    0.017    4.092    0.000
   .app                 0.300    0.029   10.176    0.000
   .hap                 0.198    0.022    8.912    0.000
   .md1                 0.212    0.044    4.764    0.000
   .md2                 0.197    0.020    9.871    0.000
   .wk1                 0.259    0.042    6.180    0.000
   .wk2                 0.372    0.050    7.454    0.000
   .wk3                 0.382    0.044    8.647    0.000
    Construc            0.215    0.050    4.263    0.000
   .Dysfunc             0.232    0.031    7.600    0.000
   .WellBe              0.090    0.020    4.459    0.000
   .JobSat              0.471    0.067    7.036    0.000
```

The implication is that constructive thinking reduces dysfunctional thinking, dysfunctional thinking reduces subjective well-being and job satisfaction, and subjective well-being increases job satisfaction.

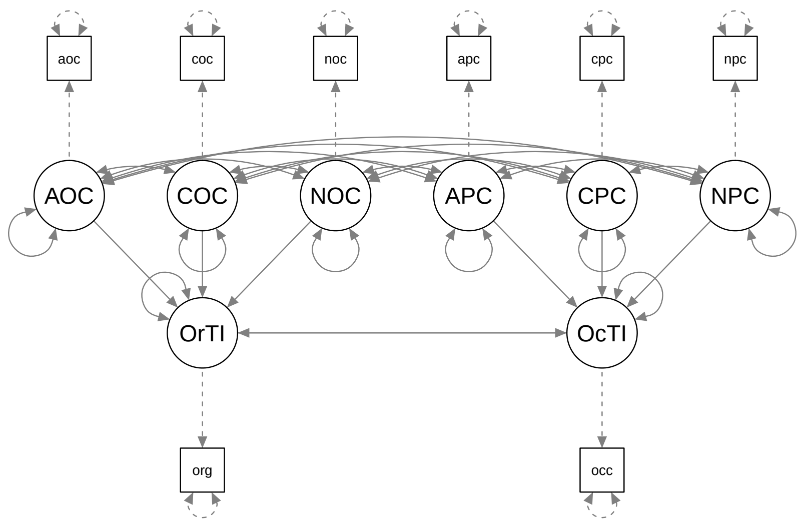
Single indicator, non-recursive model

This example involves a study of 177 nurses intended to investigate occupational commitment (ie, to the nursing profession), organizational commitment (ie, to the hospital that currently employs them) and turnover intention. There are three types of organizational and occupational commitment: affective (emotional attachment), continuance (perceived cost of leaving) and normative (feeling of obligation to stay), and two types of turnover intention (occupational and organizational). The hypothesis is that the three types of occupational commitment influence occupational turnover intention, the three types of organizational commitment influence organizational turnover intention, and organizational and occupational turnover intention mutually influence each other.

Normally, with only one indicator per factor, we would not be able to estimate error variances, but the original authors also published score reliability coefficients, which we can use to calculate measurement error variance (see p. 222 of Kline).

A diagramme of the model is shown below:

```
          aoc     coc     noc     apc     cpc    npc   orgti   occti
aoc    1.0816 -0.1019  0.6658  0.5341  0.0649  0.544 -0.7717 -0.7800
coc   -0.1019  0.9604  0.0951  0.0629  0.4434  0.128 -0.0549 -0.0294
noc    0.6658  0.0951  0.9409  0.4359  0.1135  0.465 -0.7876 -0.5820
apc    0.5341  0.0629  0.4359  1.1449  0.1836  0.805 -0.5093 -1.0111
cpc    0.0649  0.4434  0.1135  0.1836  0.6084  0.289 -0.1420 -0.3276
npc    0.5441  0.1282  0.4652  0.8047  0.2891  1.188 -0.5188 -0.9483
orgti -0.7717 -0.0549 -0.7876 -0.5093 -0.1420 -0.519  1.9600  1.1760
occti -0.7800 -0.0294 -0.5820 -1.0112 -0.3276 -0.948  1.1760  2.2500
```



Results

```
lavaan (0.5-23.1097) converged normally after  54 iterations

  Number of observations                           177

  Estimator                                         ML
  Minimum Function Test Statistic                9.420
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.051

Model test baseline model:

  Minimum Function Test Statistic              627.135
  Degrees of freedom                                28
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.991
  Tucker-Lewis Index (TLI)                       0.937

Loglikelihood and Information Criteria:
```

```
  Loglikelihood user model (H0)              -1808.897
  Loglikelihood unrestricted model (H1)      -1804.187

  Number of free parameters                         32
  Akaike (AIC)                                3681.794
  Bayesian (BIC)                              3783.431
  Sample-size adjusted Bayesian (BIC)         3682.093

Root Mean Square Error of Approximation:

  RMSEA                                          0.087
  90 Percent Confidence Interval          0.000  0.161
  P-value RMSEA <= 0.05                          0.159

Standardized Root Mean Square Residual:

  SRMR                                           0.018

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  AOC =~
    aoc               1.000
  COC =~
    coc               1.000
  NOC =~
    noc               1.000
  APC =~
    apc               1.000
  CPC =~
    cpc               1.000
  NPC =~
    npc               1.000
  OrgTI =~
    orgti             1.000
  OccTI =~
    occti             1.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  OrgTI ~
    AOC               0.052    0.376    0.137    0.891
    COC               0.044    0.171    0.255    0.799
    NOC              -1.162    0.403   -2.882    0.004
  OccTI ~
    APC              -0.658    0.184   -3.574    0.000
    CPC              -0.102    0.156   -0.649    0.516
    NPC              -0.291    0.204   -1.424    0.154
  OrgTI ~
```

```
    OccTI             0.031    0.141    0.222    0.824
  OccTI ~
    OrgTI             0.224    0.119    1.876    0.061

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .OrgTI ~~
   .OccTI             0.274    0.173    1.591    0.112
  AOC ~~
    COC              -0.101    0.077   -1.322    0.186
    NOC               0.661    0.090    7.319    0.000
    APC               0.540    0.092    5.845    0.000
    CPC               0.066    0.061    1.084    0.279
    NPC               0.546    0.094    5.804    0.000
  COC ~~
    NOC               0.092    0.071    1.292    0.196
    APC               0.052    0.078    0.663    0.507
    CPC               0.439    0.066    6.647    0.000
    NPC               0.122    0.080    1.520    0.129
  NOC ~~
    APC               0.432    0.084    5.144    0.000
    CPC               0.115    0.057    2.027    0.043
    NPC               0.458    0.086    5.349    0.000
  APC ~~
    CPC               0.187    0.064    2.924    0.003
    NPC               0.799    0.106    7.550    0.000
  CPC ~~
    NPC               0.290    0.067    4.309    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .aoc               0.195
   .coc               0.288
   .noc               0.245
   .apc               0.160
   .cpc               0.176
   .npc               0.190
   .orgti             0.274
   .occti             0.270
    AOC               0.881    0.114    7.704    0.000
    COC               0.667    0.102    6.570    0.000
    NOC               0.693    0.100    6.960    0.000
    APC               0.977    0.121    8.082    0.000
    CPC               0.430    0.064    6.672    0.000
    NPC               0.990    0.125    7.892    0.000
   .OrgTI             0.753    0.189    3.989    0.000
   .OccTI             0.669    0.127    5.271    0.000
      aoc    coc    noc    apc    cpc    npc    orgti  occti
aoc       NA
coc       NA     NA
noc       NA  0.514     NA
apc       NA  2.027  0.293  0.159
cpc       NA  1.858 -0.429     NA     NA
```

```
npc       NA  1.221  0.403  0.301     NA  0.238
orgti -1.113  1.565  0.485 -0.166 -0.798  0.382     NA
occti -1.811  2.665  1.876  0.151 -2.064  2.212     NA  0.000
```

```
      lhs op   rhs   mi    epc sepc.lv sepc.all sepc.nox
124   apc ~~   npc 8.54  0.904   0.904    0.780    0.780
59    COC =~   cpc 8.36  9.495   7.755    9.957    9.957
60    COC =~   npc 8.36  3.316   2.709    2.494    2.494
58    COC =~   apc 8.36  1.467   1.198    1.123    1.123
149 OccTI  ~   COC 8.36  0.965   0.562    0.562    0.562
62    COC =~ occti 8.36  0.965   0.788    0.527    0.527
61    COC =~ orgti 8.36 -4.306  -3.517   -2.519   -2.519
144   NPC ~~ OccTI 7.99  0.621   0.445    0.445    0.445
22    npc ~~   npc 7.99  2.136   2.136    1.810    1.810
131   npc ~~ occti 7.66  0.603   0.603    0.371    0.371
```

The largest standadized residual concerns the relationship between occupational turnover intention and continuance organizational commitment. This also has one of the largest modification indices, so try adding that to the model.

```
# respecified model with single indicators
chang.model2 <- '

#latent variables
 AOC =~ aoc
 COC =~ coc
 NOC =~ noc
 APC =~ apc
 CPC =~ cpc
 NPC =~ npc
 OrgTI =~ orgti
 OccTI =~ occti

#regressions
 OrgTI ~ AOC + COC + NOC
 OccTI ~ APC + CPC + NPC + COC
 OrgTI ~ OccTI
 OccTI ~ OrgTI

#fix error variances
 aoc ~~ .1947*aoc
 coc ~~ .2881*coc
 noc ~~ .2446*noc
 apc ~~ .1603*apc
 cpc ~~ .1764*cpc
 npc ~~ .1901*npc
 orgti ~~ .2744*orgti
 occti ~~ .2700*occti

#correlated disturbances
 OrgTI ~~ OccTI '

# fit respecified model to data
model2 <- sem(chang.model2,
       sample.cov = changFull.cov,
```

```r
       sample.nobs = 177)
```

Compare model fits:

```r
anova(model1, model2)
```

```
Chi Square Difference Test

       Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
model2  3 3675 3780  0.81
model1  4 3682 3783  9.42       8.61       1     0.0033
```

```r
summary(model2, fit.measure = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  63 iterations

  Number of observations                           177

  Estimator                                         ML
  Minimum Function Test Statistic                0.809
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.847

Model test baseline model:

  Minimum Function Test Statistic              627.135
  Degrees of freedom                                28
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.034

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1804.592
  Loglikelihood unrestricted model (H1)      -1804.187

  Number of free parameters                         33
  Akaike (AIC)                                3675.184
  Bayesian (BIC)                              3779.997
  Sample-size adjusted Bayesian (BIC)         3675.492

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.070
  P-value RMSEA <= 0.05                          0.913

Standardized Root Mean Square Residual:

  SRMR                                           0.005

Parameter Estimates:
```

```
  Information                                      Expected
  Standard Errors                                  Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  AOC =~
    aoc               1.000
  COC =~
    coc               1.000
  NOC =~
    noc               1.000
  APC =~
    apc               1.000
  CPC =~
    cpc               1.000
  NPC =~
    npc               1.000
  OrgTI =~
    orgti             1.000
  OccTI =~
    occti             1.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  OrgTI ~
    AOC              -0.062    0.413   -0.149    0.882
    COC               0.030    0.174    0.170    0.865
    NOC              -1.033    0.408   -2.534    0.011
  OccTI ~
    APC              -0.754    0.231   -3.259    0.001
    CPC              -1.441    0.647   -2.226    0.026
    NPC               0.094    0.309    0.306    0.760
    COC               0.996    0.454    2.195    0.028
  OrgTI ~
    OccTI             0.037    0.147    0.250    0.803
  OccTI ~
    OrgTI             0.287    0.145    1.982    0.047

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .OrgTI ~~
   .OccTI             0.226    0.178    1.272    0.203
  AOC ~~
    COC              -0.104    0.076   -1.362    0.173
    NOC               0.661    0.090    7.315    0.000
    APC               0.532    0.092    5.768    0.000
    CPC               0.067    0.061    1.103    0.270
    NPC               0.541    0.094    5.754    0.000
  COC ~~
    NOC               0.092    0.071    1.295    0.195
    APC               0.063    0.078    0.808    0.419
    CPC               0.443    0.066    6.733    0.000
    NPC               0.127    0.080    1.587    0.112
```

```
  NOC ~~
    APC               0.431    0.084    5.136    0.000
    CPC               0.116    0.057    2.039    0.041
    NPC               0.458    0.086    5.334    0.000
  APC ~~
    CPC               0.182    0.064    2.856    0.004
    NPC               0.800    0.106    7.556    0.000
  CPC ~~
    NPC               0.288    0.067    4.289    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .aoc               0.195
   .coc               0.288
   .noc               0.245
   .apc               0.160
   .cpc               0.176
   .npc               0.190
   .orgti             0.274
   .occti             0.270
    AOC               0.881    0.114    7.704    0.000
    COC               0.664    0.101    6.581    0.000
    NOC               0.695    0.100    6.963    0.000
    APC               0.978    0.121    8.083    0.000
    CPC               0.427    0.064    6.670    0.000
    NPC               0.991    0.126    7.894    0.000
   .OrgTI             0.767    0.192    4.007    0.000
   .OccTI             0.461    0.157    2.933    0.003
```

Homework

```
      EdAsp OcAsp VerbAch QuantAch FamInc  FaEd  MoEd VerbAb QuantAb
 [1,] 1.024 0.000   0.000    0.000  0.000 0.000 0.000  0.000   0.000
 [2,] 0.792 1.077   0.000    0.000  0.000 0.000 0.000  0.000   0.000
 [3,] 1.027 0.919   1.844    0.000  0.000 0.000 0.000  0.000   0.000
 [4,] 0.756 0.697   1.244    1.286  0.000 0.000 0.000  0.000   0.000
 [5,] 0.567 0.537   0.876    0.632  0.852 0.000 0.000  0.000   0.000
 [6,] 0.445 0.424   0.677    0.526  0.518 0.670 0.000  0.000   0.000
 [7,] 0.434 0.389   0.635    0.498  0.475 0.545 0.716  0.000   0.000
 [8,] 0.580 0.564   0.893    0.716  0.546 0.422 0.373  0.851   0.000
 [9,] 0.491 0.499   0.888    0.646  0.508 0.389 0.339  0.629   0.871
```

The model is that a student's achievement (`Achieve`) depends on home and family characteristics (`Home`), the student's ability (`Ability`), and the student's aspiration (`Aspire`). These four latent variable have the following measures:

Achieve: Verbal achievement (VerbAch) and Quantitative achievement (QuantAch). *Home*: Family income (FamInc), Father's education (FaEd) and Mother's education (MoEd). *Ability*: Verbal ability (VerbAb) and Quantitative ability (QuantAb). *Aspire*: Educational aspiration (EdAsp) and Occupational aspiration (OcAsp)

The covariance matrix above is based on data from 200 students.

Aspiration is thought to be influenced by Ability and Home situation. Achievement is thought to be influenced by all three latent variables. Specify an appropriate structural equation model, obtain results and determine whether the fit of the model is adequate. If it is not, modify the model and re-fit.

Since we're not having another class, I've included the answers, but try to do it yourself first!

First SEM

```
lavaan (0.5-23.1097) converged normally after  43 iterations

  Number of observations                           200

  Estimator                                         ML
  Minimum Function Test Statistic               57.454
  Degrees of freedom                                21
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Home =~
    FamInc            1.000
    FaEd              1.007    0.073   13.725    0.000
    MoEd              0.964    0.076   12.707    0.000
  Ability =~
    VerbAb            1.000
    QuantAb           0.949    0.068   14.017    0.000
  Aspire =~
    EdAsp             1.000
    OcAsp             0.917    0.064   14.341    0.000
  Achieve =~
    VerbAch           1.000
    QuantAch          0.759    0.042   18.206    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Aspire ~
    Home              0.410    0.125    3.282    0.001
    Ability           0.590    0.116    5.081    0.000
  Achieve ~
    Home              0.242    0.128    1.901    0.057
    Ability           0.751    0.142    5.298    0.000
    Aspire            0.548    0.113    4.859    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  Home ~~
    Ability           0.429    0.062    6.877    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .FamInc            0.318    0.039    8.106    0.000
```

```
   .FaEd              0.129    0.024    5.303    0.000
   .MoEd              0.221    0.030    7.394    0.000
   .VerbAb            0.187    0.035    5.328    0.000
   .QuantAb           0.273    0.038    7.146    0.000
   .EdAsp             0.159    0.041    3.854    0.000
   .OcAsp             0.349    0.047    7.370    0.000
   .VerbAch           0.204    0.051    4.026    0.000
   .QuantAch          0.340    0.043    7.866    0.000
    Home              0.530    0.082    6.469    0.000
    Ability           0.660    0.088    7.518    0.000
   .Aspire            0.333    0.058    5.781    0.000
   .Achieve           0.224    0.057    3.911    0.000
```

```
 chisq     df pvalue  rmsea    cfi   srmr
57.454 21.000  0.000  0.093  0.974  0.048
```

```
       lhs op     rhs    mi    epc sepc.lv sepc.all sepc.nox
64    FaEd ~~    MoEd 40.38  0.204   0.204    0.296    0.296
49 Achieve =~  FamInc 35.95  0.413   0.527    0.572    0.572
35 Ability =~  FamInc 35.63  0.620   0.504    0.547    0.547
42  Aspire =~  FamInc 23.04  0.401   0.371    0.403    0.403
57  FamInc ~~    MoEd 10.55 -0.098  -0.098   -0.126   -0.126
50 Achieve =~    FaEd 10.04 -0.206  -0.263   -0.322   -0.322
37 Ability =~    MoEd  9.88 -0.298  -0.242   -0.287   -0.287
56  FamInc ~~    FaEd  7.93 -0.090  -0.090   -0.120   -0.120
80  VerbAb ~~ VerbAch  7.71 -0.086  -0.086   -0.069   -0.069
43  Aspire =~    FaEd  7.34 -0.203  -0.188   -0.230   -0.230
```

```
         FamInc FaEd   MoEd   VerbAb QuntAb EdAsp  OcAsp  VrbAch QntAch
FamInc    0.000
FaEd         NA  0.000
MoEd         NA  2.173  0.000
VerbAb    3.334 -1.064 -2.835  0.000
QuantAb   2.784 -1.537 -2.994  0.000     NA
EdAsp     2.726 -4.472 -1.170  0.749 -4.728     NA
OcAsp     2.582 -0.616 -1.125  1.610  0.182     NA     NA
VerbAch   3.468     NA -3.416     NA  1.186  0.875 -1.178  0.000
QuantAch  2.290 -0.979 -0.919  1.061 -0.637 -0.951 -0.384  0.000  0.000
```

There are a number of possible modifications that these results suggest. Examples include covariance between father's and mother's education (which seems very plausible). Let's try this out.

```
lavaan (0.5-23.1097) converged normally after  46 iterations

  Number of observations                           200

  Estimator                                         ML
  Minimum Function Test Statistic               19.265
  Degrees of freedom                                20
  P-value (Chi-square)                           0.505

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Home =~
    FamInc            1.000
    FaEd              0.782    0.064   12.215    0.000
    MoEd              0.720    0.069   10.398    0.000
  Ability =~
    VerbAb            1.000
    QuantAb           0.949    0.067   14.136    0.000
  Aspire =~
    EdAsp             1.000
    OcAsp             0.918    0.064   14.377    0.000
  Achieve =~
    VerbAch           1.000
    QuantAch          0.753    0.041   18.180    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Aspire ~
    Home              0.506    0.153    3.297    0.001
    Ability           0.447    0.151    2.968    0.003
  Achieve ~
    Home              0.302    0.161    1.879    0.060
    Ability           0.685    0.160    4.277    0.000
    Aspire            0.526    0.115    4.567    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  Home ~~
    Ability           0.535    0.070    7.656    0.000
 .FaEd ~~
   .MoEd              0.172    0.032    5.290    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .FamInc            0.189    0.040    4.755    0.000
   .FaEd              0.264    0.034    7.680    0.000
   .MoEd              0.371    0.044    8.519    0.000
   .VerbAb            0.187    0.035    5.420    0.000
   .QuantAb           0.273    0.038    7.213    0.000
   .EdAsp             0.160    0.041    3.894    0.000
   .OcAsp             0.348    0.047    7.376    0.000
   .VerbAch           0.192    0.050    3.817    0.000
   .QuantAch          0.347    0.044    7.965    0.000
    Home              0.658    0.090    7.337    0.000
    Ability           0.659    0.088    7.533    0.000
   .Aspire            0.317    0.056    5.621    0.000
   .Achieve           0.227    0.057    3.984    0.000

 chisq pvalue  rmsea    cfi   srmr
19.265  0.505  0.000  1.000  0.015
```

That seems to do the trick. You can see that all the hypothesised structural relationships are statistically significant.